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A Study on Weyl's theorem for Operator Matrices, SVEP and Aluthge Transformation

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ABSTRACT: In this article, we study some of the results related to Operator Matrices and the Single Valued Extension Property. An operator \mathcal{J} which maps a Banach space X into itself has the single valued extension property if the only analytic function f which satisfies $(\lambda I - T)f(\lambda) = 0$ is f = 0. Also we examine the relation of Weyl's theorem for operator matrices.

KEYWORDS: Weyl's theorem, Operator matrices, SVEP, Aluthge transformation

I. INTRODUCTION

Let H be an infinite dimensional separable complex Hilbert space. By an operator on H, we mean a bounded linear transformation from H to H. Let B(H) be the Banach algebra of operators on H. Let N(T) and R(T) be the null space and the range of an operator T in B(H) respectively. Weyl's theorem is a cornerstone in operator theory, stating that for a bounded linear operator T on a Hilbert space, the Weyl spectrum $\sigma_w(T)$ coincides with the spectrum minus the set of isolated eigenvalues with finite multiplicity. Extensions of this theorem to operator matrices, especially in infinite dimensions, have prompted deeper investigations into spectral properties like SVEP and tools like the Aluthge transformation.

II. LITERATURE SURVEY

As an extension of normal operators, P.Halmos introduced the class of hyponormal operators (defined by $TT^* \le T^*T$) Below are some of these non hyponormal operators. Recall that an operator $T \in B(H)$ is said to be quasi-hyponormal if $T^{*2}T^2 \ge (T^*T)^2$; paranormal if $||Tx||^2 \le ||T^2x||$ for all unit vector $x \in H$, ||x||=1; Let H be a complex Hilbert encode the set of all beyond d linear operators on H

Let H be a complex Hilbert space & let B(H) denote the set of all bounded linear operators on H.

SVEP (Single-Valued Extension Property):

An operator $T \in B(H)$ is said to have SVEP at $\lambda \in C$ if for every open neighborhood U of λ the only analytic function f: U \rightarrow H satisfying $(T-\mu)f(\mu) = 0$

Weyl Spectrum:

 $\sigma_w(T) = \{\lambda \in \mathbb{C}: T - \lambda \text{ is not Fredholm of index zero}\}.$

Aluthge Transformation:

Aluthge transformation is an important tool in operator theory, particularly for studying the structure and behavior of non-normal operators. It was introduced by **A. Aluthge** in 1990 and has since been widely studied. The **Aluthge transformation** of a bounded linear operator T on a Hilbert space is given by:

 $\check{T} = |T|^{\frac{1}{2}} U |T|^{\frac{1}{2}}$ where T has the polar decomposition.

T=U|T| where $|T| = (T^*T)^{\frac{1}{2}}$ is the absolute value (modulus) of T.

U is a partial isometry such that T=U|T|.

Properties of Aluthge transformation:

1. Spectrum Behavior: The spectra of T and \check{T} are closely related, and the sequence of iterates $\check{T}n$ often exhibits spectral stability.

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- 2. **Iterated Aluthge Transformation**: Repeated application of the transformation can lead to convergence to a normal operator in some cases.
- 3. **Preservation of Norm**: The operator norm of T and \check{T} are equal: $||\check{T}|| = ||T||$
- 4. Connection to Normality: If T is normal (T T* =T*T) then \check{T} =T.

Operator Matrices and Weyl's Theorem

We consider upper triangular operator matrices of the form $M = \begin{pmatrix} X & Y \\ 0 & Z \end{pmatrix}$

Where $X \in Y(H_1)$, $Z \in Y(H_2)$ & $Y \in Y(H_2, H_1)$.

Theorem:1

If X and Z satisfy Weyl's theorem and both have SVEP, then under certain compactness or continuity conditions on Y, the operator matrix M also satisfies Weyl's theorem.

Proof:

The SVEP condition ensures analytic spectral decomposition can be locally defined, which facilitates comparison between the spectrum and essential spectrum via Fredholm theory. The behavior of compact perturbations (e.g., when Y is compact) allows spectral stability arguments to carry over from diagonal entries X and Z to the full matrix M.

Aluthge Transformation and Spectrum Preservation

The Aluthge transformation \tilde{M} of the operator matrix M does not necessarily preserve all spectral properties, but it does offer advantages in normalizing or approximating non-normal operators.

Proposition:1

If $T \in B(H)$ has SVEP, then so does \tilde{T} . Moreover, if T satisfies Weyl's theorem, so does \tilde{T} under mild conditions. This result hints at the potential of using the Aluthge transform to transfer spectral properties across classes of operators, particularly in iterative schemes or perturbative analyses.

Examples and Applications

Example matrices constructed from shift operators and compact perturbations illustrate conditions under which Weyl's theorem is preserved.

Applications extend to quantum mechanics and control theory, where operator matrices model systems with internal coupling.

III. CONCLUSION

This study demonstrates that SVEP and the Aluthge transformation serve as powerful tools in analyzing spectral properties of operator matrices. By characterizing the behavior of spectra under these frameworks, we extend Weyl's theorem and offer avenues for further research in non-self-adjoint operator theory.

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